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AD-A204 640 DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

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1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)
AFOSR-TR- 89-0018

6a. NAME OF PERFORMING ORGANIZATION
Brown University

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION
AFOSR/NM

6c. ADDRESS (City, State, and ZIP Code)
Providence, RI

7b. ADDRESS (City, State, and ZIP Code)
Building 410
Bolling, AFB DC 20332-6448

8a. NAME OF FUNDING / SPONSORING ORGANIZATION
AFOSR

8b. OFFICE SYMBOL (If applicable)
NM

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR 84-0398

8c. ADDRESS (City, State, and ZIP Code)
Building 410
Bolling, AFB DC 20332-6448

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A1	

11. TITLE (Include Security Classification)
Computational Methods for Control & Estimation of Distributed Systems

12. PERSONAL AUTHOR(S)
H.T Banks

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
FINAL	FROM 9/84 TO 2/88	880801	74

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research concentrated in two areas, control and parameter estimation. In the control area, results were obtained in the development, analysis, and numerical testing of computational techniques. In the parameter estimation and inverse problems area, results includes thoerectical frameworks for stability and convergence of approximation schemes, regulation techniques, augmented Lograngian methods, and estimation of nonlinearities.

This document contains title pages and abstracts only for a large number of publications.
Keywords: applied mathematics. (KR)

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13 FEB 1989
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20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☑ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
Unclass

22a. NAME OF RESPONSIBLE INDIVIDUAL
Lt Col James M. Crowley

22b. TELEPHONE (Include Area Code)
(202) 767- 5025

22c. OFFICE SYMBOL
NM

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

AFOSR-TR- 89-0018

AIR FORCE FINAL TECHNICAL REPORT

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Justification	
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Availability Codes	
Dist	Avail and/or Special
A-1	

Grant No.
AFOSR - 84 - 0398

For the period:
September 1984 - February 1988
3 years and 6 months' extension

Prepared by: H.T. Banks
August 1, 1988

89 2 10 115

Summary of Research Effort

The research efforts supported by this grant have two separate but closely related thrusts: (i) Control and (ii) parameter estimation (inverse problems) for distributed parameter systems including delay and partial differential equations. A diverse number of topics have been investigated and can be summarized as follows.

I. *Control* (the main emphasis was the development, analysis, and numerical testing of computational techniques):

(i) Numerical methods for computation of feedback gains in high dimensional LQR problems (a hybrid method: Chandrasekhar start up; modified Kleinman-Newton-Smith iteration for the gains).

(ii) Approximation techniques for finite dimensional compensators in infinite dimensional feedback control problems.

(iii) Approximate solutions of operator Riccati equations: regularity, convergence.

(iv) Feedback control techniques for infinite dimensional systems.

(v) Control and stabilization of visco-elastic structures.

(vi) Approximation in delay and Volterra type equations.

II. *Parameter estimation and inverse problems* (with emphasis on development and analysis of computational techniques for specific applications in mechanics and biology):

(i) Theoretical frameworks for stability and convergence of approximation schemes: variational and semigroup formulations.

(ii) Regularization techniques: Compactness, discretization, Tikhonov techniques.

(iii) Augmented Lagrangian methods.

(iv) Estimation of damping in flexible structures: composite and viscoelastic materials.

(v) Estimation of growth, predation, dispersal, size-structure in population models.

(vi) Estimation of nonlinearites in distributed systems.

Air Force Final Technical Report

Grant No. AFOSR - 84 - 0398

September 1984 - February 1988
3 years and 6 months' extension

H.T. Banks

[1985, May] On a Variational Approach to Some Parameter Estimation Problems.

[1986, January] A Comparison of Stability and Convergence Properties of Techniques for Inverse Problems.

[1986, January] Quantitative Modeling of Growth and Dispersal in Population Models.

[1986, July 2] Estimation of Stiffness and Damping in Cantilevered Euler-Bernoulli Beams with Tip Bodies.

[1986, August] Computational Methods for the Identification of Spatially Varying Stiffness and Damping in Beams.

[1986, August] The Identification of a Distributed Parameter Model for a Flexible Structure.

[1986, October] A Numerical Algorithm for Optimal Feedback Gains in High Dimensional LQR Problems.

[1986, November] Analyzing Field Studies of Insect Dispersal Using Two-Dimensional Transport Equations

[1987, January] Inverse Problems in the Modeling of Vibrations of Flexible Beams.

[1987, March] Modeling and Estimation in Size Structured Population Models.

[1987, March] A Theoretical Framework for convergence and Continuous Dependence of Estimates in Distributed Parameter Systems.

[1987, May] Parameter estimation techniques for interaction and redistribution models: a predator-prey example.

[1987, August] Estimation of Nonlinearities in Parabolic Models for Growth, Predation and Dispersal of Populations.

[1987, October] Computational Techniques for Estimation and Control of Distributed Parameter Systems.

[1987, December] Parameter Identification Techniques for the Estimation of Damping in Flexible Structure Experiments.

[1988, April] An Approximation Theory for the Identification of Nonlinear Distributed Parameter Systems.

R.H. Fabiano

[1988, May] Semigroup Theory and Numerical Approximation for Equations in Linear Viscoelasticity.

K. Ito

[1986, April] The Augmented Lagrangian Method for Equality and Inequality Constraints in Hilbert Spaces.

[1986, May] On the Regularity of Solutions of an Operator Riccati Equation Arising in Linear Quadratic Optimal Control Problems for Hereditary Differential Systems.

[1986, September] Approximation of Infinite Delay and Volterra Type Equations.

[1986, December] Finite Dimensional Compensators for Infinite Dimensional Systems Via Galerkin-Type Approximation.

[1987, April] The Augumented Langrangian Method for Parameter Estimation in Elliptic Systems.

[1987, April] Chandrasekar Equations for Infinite Dimensional Systems: Part II. Unbounded Input and Output Case.

[1987, April] Strong Convergence and Convergence Rates of Approximating Solutions for Algebraic Riccati Equations in Hilbert Spaces.

[1987, May] Chandrasekhar Equations for Infinite Dimensional Systems.

[1987, June] A Uniformly Differentiable Approximation Scheme for Delay Systems Using Splines.

[1987, July] On Non-Convergence of Adjoint Semigroups for Control Systems with Delays.

F. Kappel

[1986, September] Approximation of Infinite Delay and Volterra Type Equations.

[1987, June] A Uniformly Differentiable Approximation Scheme for Delay Systems Using Splines.

K. Kunisch

[1986, April] The Augmented Lagrangian Method for Equality and Inequality Constraints in Hilbert Spaces.

[1987, April] The Augumented Langrangian Method for Parameter Estimation in Elliptic Systems.

K.A. Murphy

[1986, January] Quantitative Modeling of Growth and Dispersal in Population Models.

[1987, May] Parameter estimation techniques for interaction and redistribution models: a predator-prey example.

[1987, August] Estimation of Nonlinearities in Parabolic Models for Growth, Predation and Dispersal of Populations.

ON A VARIATIONAL APPROACH
TO SOME PARAMETER ESTIMATION PROBLEMS*

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May, 1985

* Invited Lecture, International Conference on Control Theory for Distributed Parameter Systems and Applications, July 9-14, 1984, Vorau, Austria.

+ The research reported here was supported in part by NSF grant DMS 8205355, by AFOSR contract AF-AFOSR 84-0398, and ARO contract DAAG 29-83-K-0029. Parts of the research were carried out while the author was a visitor at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA, which is operated under NASA contract NAS1-17070.

On a Variational Approach to Some Parameter Estimation Problems

ABSTRACT

We consider examples (1-D seismic, large flexible structures, bioturbation, nonlinear population dispersal) in which a variational setting can provide a convenient framework for convergence and stability arguments in parameter estimation problems.

A Comparison of Stability and

Convergence Properties of Techniques

for Inverse Problems*

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January 1986

* This research was supported in part by the National Science Foundation under NSF Grant MCS-8504316, the Air Force Office of Scientific Research under Contract AFOSR-84-0398, and the National Aeronautics and Space Administration under NASA Grant NAG-1-517.

\+ Part of this research was carried out while the first author was a visiting scientist at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA, which is operated under NASA Contracts NAS1-17070 and NAS1-18107.

Abstract

We consider a series of numerical examples and compare several algorithms for estimation of coefficients in differential equation models. Unconstrained, constrained and Tikhonov regularization methods are tested for the behavior with regard to both convergence (of approximation methods for the states and parameters) and stability (continuity of the estimates obtained with respect to perturbations in the data or observed states).

QUANTITATIVE MODELING OF GROWTH AND DISPERSAL IN POPULATION MODELS

H.T. Banks and K.A. Murphy

January, 1986 LCDS #86-4

QUANTITATIVE MODELING OF GROWTH

AND DISPERSAL IN POPULATION MODELS*

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ABSTRACT

We discuss techniques for the estimation of nonlinearities and state-dependent coefficients in parabolic partial differential equations. Applications to density-dependent population dispersal and nonlinear growth/predation models are presented. Computational results using parallel and vector architectures are discussed.

* Lecture presented at the International Symposium on Mathematical Biology, November 10-15, 1985, Kyoto, Japan.

+ Research supported in part by the National Science Foundation under NSF Grant MCS-8504316, the Air Force Office of Scientific Research under Contract No. AFOSR-84-0398, and the Army Research Office under Contract No. ARO-DAA6-29-83-K-0029. Part of the research was carried out while the authors were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665, which is operated under NASA Contract NAS1-17070.

QUANTITATIVE MODELING OF GROWTH AND DISPERSAL IN POPULATION MODELS*

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ABSTRACT

We discuss techniques for the estimation of nonlinearities and state-dependent coefficients in parabolic partial differential equations. Applications to density-dependent population dispersal and nonlinear growth/predation models are presented. Computational results using parallel and vector architectures are discussed.

* Lecture presented at the International Symposium on Mathematical Biology, November 10-15, 1985, Kyoto, Japan.

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Proc. 4th IFAC Symposium on Control of Distributed Parameter Systems, June 30-July 2, 1986, Los Angeles, CA.

ESTIMATION OF STIFFNESS AND DAMPING IN CANTILEVERED EULER-BERNOULLI BEAMS WITH TIP BODIES

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Abstract. We develop finite dimensional approximation schemes for the identification of spatially varying material parameters, i.e. flexural stiffness and viscous damping coefficients in hybrid models for flexible beams with tip bodies. Our schemes are derived via an application of spline-based Galerkin techniques to the conservative form state space representation for the coupled system of ordinary and partial differential equations and boundary conditions which describe the dynamics of the system. A convergence theory is briefly outlined and a discussion of our findings based upon extensive numerical studies carried out on both conventional and vector processors is included.

Keywords. Parameter estimation; flexible structures; damping; approximation theory; computational methods; splines.

We report here on a part of our investigations on methods to estimate continuum material parameters, and specifically stiffness and damping parameters in structures described by an Euler - Bernoulli formulation of the dynamics. Our efforts have been motivated in large part by research on large flexible space structures. Among such space structures of interest are plate and beam-like truss structures, space platforms with multiple appendages including solar panels, robot arms and masts with tip bodies (e.g. the shuttle orbiter with beam/tip mass configurations) and large antennas. These structures are highly flexible, possess relatively low damping characteristics, and in general won't support their own weight in earth's gravity. Their flexible members (truss beams, panels) are usually constructed of composite materials (e.g. graphite epoxy) while membrane-like woven mesh surfaces are found in the antenna reflectors. Thus, in these investigations one must deal with complex composite structures with variable (in space and time) geometry and material properties for which continuum models with variable parameters offer some obvious advantages.

The identification or parameter estimation techniques which we describe here are important for model development and analysis, periodic material parameter updates in structures, as well as being a precursor to and integral part of the development of sophisticated control and stabilization strategies for large flexible structures. We focus here on the possibilities of estimating viscoelastic damping of Voigt-Kelvin type (Clough and Penzien, 1975; Popov, 1968) - i.e. where the damping moment is proportional to the strain rate. While there is growing evidence that damping in composite material structures is significantly more complex then that arising from the Voigt-Kelvin hypothesis, there are materials for which the Voigt-Kelvin assumption is a good approximation. Therefore such models provide an excellent class on which to start in development and testing of methods.

In addition to our investigations on approximation methods for estimation problems, we have also been exploring the use of special architectures-array processors and vector machines - in conjunction with the methods and algorithms we are developing. A brief discussion of certain aspects of our findings in this area is included below.

The results presented here are based on a conservative state space form of the dynamical equations for a cantilevered beam with tip body attached to the free end. That is, we consider the higher order analogue of the classical formulation in which a second order (in time and space derivatives) hyperbolic equation is rewritten as a first order vector system of equations by choosing the natural states of strain u_x and velocity u_t. Due to space limitations, we shall attempt to keep details, discussions, and notation to a minimum. For more complete and careful discussions we refer readers to (Banks and Rosen, 1985 and 1986).

As we have indicated above, the structures of interest are in general quite complex. The simplest problems involve beam and plate-like models or models for membrane-like surfaces. One popular and useful class of dynamical system models involves a cantilevered beam with tip body and base acceleration. Such models are useful, for example, in studying shuttle deployed masts with attached payload - e.g. antenna - as

(*)Research supported in part by the National Science Foundation under NSF Grant MCS-8504316 and the Air Force Office of Scientific Research under Contract No. AFOSR-84-0398.

(1)Research supported in part by the Air Force Office of Scientific Research under Contract No. AFOSR-84-0393

Part of the research was carried out while the first two authors were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665 which is operated under NASA Contract NAS1 - 18107.

COMPUTATIONAL METHODS FOR THE IDENTIFICATION OF SPATIALLY VARYING STIFFNESS AND DAMPING IN BEAMS+

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** This research was supported in part by the Air Force Office of Scientific Research under Contract AFOSR-84-0393.

+ Part of this research was carried out while the authors were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665-5225, which is operated under NASA Contracts NAS1 - 17070 and NAS1 - 18107.

Computational Methods for the Identification of Spatially Varying Stiffness and Damping in Beams

ABSTRACT

A numerical approximation scheme for the estimation of functional parameters in Euler-Bernoulli models for the tansverse vibration of flexible beams with tip bodies is developed. The method permits the identification of spatially varying flexural stiffness and Voigt-Kelvin viscoelastic damping coefficients which appear in the hybrid system of ordinary and partial differential equations and boundary conditions describing the dynamics of such structures. An inverse problem is formulated as a least squares fit to data subject to constraints in the form of a vector system of abstract first order evolution equations. Spline-based finite element approximations are used to finite dimensionalize the problem. Theoretical convergence results are given and numerical studies carried out on both conventional (serial) and vector computers are discussed.

THE IDENTIFICATION OF A DISTRIBUTED PARAMETER MODEL FOR A FLEXIBLE STRUCTURE

by

H.T. Banks, S.S. Gates, I.G. Rosen, Y. Wang

August 1986 LCDS #86-32

ABSTRACT

We develop a computational method for the estimation of parameters in a distributed model for a flexible structure. The structure we consider (part of the "RPL experiment") consists of a cantilevered beam with a thruster and linear accelerometer at the free end. The thruster is fed by a pressurized hose whose horizontal motion effects the transverse vibration of the beam. We use the Euler-Bernoulli theory to model the vibration of the beam and treat the hose-thruster assembly as a lumped or point mass-dashpot-spring system at the tip. Using measurements of linear acceleration at the tip, we estimate the hose parameters (mass, stiffness, damping) and a Voigt-Kelvin viscoelastic structural damping parameter for the beam using a least squares fit to the data.

We consider spline based approximations to the hybrid (coupled ordinary and partial differential equations) system; theoretical convergence results and numerical studies with both simulation and actual experimental data obtained from the structure are presented and discussed.

A NUMERICAL ALGORITHM FOR OPTIMAL FEEDBACK GAINS IN HIGH DIMENSIONAL LQR PROBLEMS

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October 1986

Research supported in part by the National Aeronautics and Space Administration under NASA grant NAG-1-517, the Air Force Office of Scientific Research under contracts No. AFOSR-84-0398 and AFOSR-85-0303, and the National Science Foundation under NSF grant MCS-8504316. Parts of the research were carried out while the authors were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665, which is operated under NASA contracts NAS1-17070 and NAS1-18107.

A Numerical Algorithm for Optimal Feedback Gains in High Dimensional LQR Problems

H.T. Banks
K. Ito

ABSTRACT

We propose a hybrid method for computing the feedback gains in linear quadratic regulator problems. The method, which combines use of a Chandrasekhar type system with an iteration of the Newton-Kleinman form with variable acceleration parameter Smith schemes, is formulated so as to efficiently compute directly the feedback gains rather than solutions of an associated Riccati equation. The hybrid method is particularly appropriate when used with large dimensional systems such as those arising in approximating infinite dimensional (distributed parameter) control systems (e.g., those governed by delay-differential and partial differential equations). Computational advantages of our proposed algorithm over the standard eigenvector (Potter, Laub-Schur) based techniques are discussed and numerical evidence of the efficacy of our ideas presented.

Key Words: LQR problems, feedback gains, distributed parameter systems, computational algorithm, Chandrasekhar system, Newton-Kleinman scheme, Smith method.

ANALYZING FIELD STUDIES OF INSECT DISPERSAL USING TWO-DIMENSIONAL TRANSPORT EQUATIONS

by

H.T. Banks, P.M. Kareiva and L. Zia

November 1986 LCDS/CCS #86-48

Analyzing Field Studies of Insect Dispersal

Using Two-Dimensional Transport Equations

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proceeds in a heuristic and iterative fashion, much like an experimental inquiry (see Zia 1986 for numerical test examples and details on procedures to follow).

ACKNOWLEDGEMENTS

This research was supported by NSF grant BSR 8517183 to P. Kareiva, NSF grant MCS-8504316 to H.T. Banks, and Air Force Office of Scientific Research Contract No. AFOSR-84-0398. We thank J. Bergelson and A. Power for comments on an earlier version of this manuscript.

Analyzing Field Studies of Insect Dispersal Using Two-Dimensional Transport Equations

ABSTRACT

A variety of diffusion and convection/diffusion models were fit to field mark-recapture data (from Hawkes 1974) for female cabbage root flies (Delia brassicae Bouché). By comparing the performance of different models we were able to test Hawkes' hypothesis that anemotaxis is key to Delia's discovery of Brassica crops. Whereas models lacking a convection term totally failed, models with a convection term explained 39-44% of the observed variance in Delia recapture density. The direction of the best-fit convection vector was towards Brassica and in almost perfect opposition to the prevailing winds. This suggests that Delia fly upwind in the presence of Brassica odors. The application of diffusion/convection models to insect dispersal is discussed in general, with special emphasis on parameter identification methods. These methods (which are new to the biological literature) allow one to find realistic dispersal models that describe field mark-recapture data.

INVERSE PROBLEMS IN THE MODELING OF VIBRATIONS OF FLEXIBLE BEAMS

by

H.T. Banks, R.K. Powers, and I.G. Rosen

January 1987 LCDS/CCS #87-3

INVERSE PROBLEMS IN THE MODELING OF

VIBRATIONS OF FLEXIBLE BEAMS+

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January 1987

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Part of this research was carried out while the authors were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA, which is operated under NASA Contracts NAS1-17070 and NAS1-18107.

* This research was supported in part by the National Science Foundation under NSF Grant MCS-8504316, the Air Force Office of Scientific Research under Contract AFOSR-84-0398, and the National Aeronautics and Space Administration under NASA Grant NAG-1-517.

** This research was supported in part by the Air Force Office of Scientific Research under Contract AFOSR-84-0393.

ABSTRACT

The formulation and solution of inverse problems for the estimation of parameters which describe damping and other dynamic properties in distributed models for the vibration of flexible structures is considered. Motivated by a slewing beam experiment, the identification of a nonlinear velocity dependent term which models air drag damping in the Euler-Bernoulli equation is investigated. Galerkin techniques are used to generate finite dimensional approximations. Convergence estimates and numerical results are given. The modeling of, and related inverse problems for the dynamics of a high pressure hose line feeding a gas thruster actuator at the tip of a cantilevered beam are then considered. Approximation and convergence are discussed and numerical results involving experimental data are presented.

MODELING AND ESTIMATION IN

SIZE STRUCTURED POPULATION MODELS

by

H.T. Banks, L.W. Botsford, F. Kappel, and C.Wang

March 1987 LCDS/CCS #87-13

MODELING AND ESTIMATION IN SIZE STRUCTURED POPULATION MODELS*

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March, 1987

*Invited lecture by the first author at the Research Symposium, Second Autumn Course on Mathematical Ecology, International Centre for Theoretical Phyics, Trieste, Italy, December 8-12, 1986.

ACKNOWLEDGEMENTS

The authors would like to thank Alice Fusfeld Low of the California Department of Fish and Game (CDFG) for providing the striped bass data used in some of the investigations reported above.

The research reported here was supported in part by the National Science Foundation (H.T.B. and C.W.) under NSF Grant MCS-8504316, the Air Force Office of Scientific Research (H.T.B., C.W. and F.K.) under Contracts AFOSR-84-0398 and F-49620-86-C-011, the CDFG Striped Bass Stamp Funds (L.W.B.), and the Fonds zur Förderung der Wissenschaftlichen Forschung (Austria) (F.K.) under Project No. 53206.

Modeling and Estimation in Size Structured Population Models

ABSTRACT

We report on current investigations involving both deterministic and stochastic versions of size structured population models of the McKendrick - Von Foerster type. Simulation studies that demonstrate several mechanisms of size dispersion are presented. We also present preliminary computational results for inverse problems involving the estimation of survival and growth parameters for larval fish from size structured field data.

A THEORETICAL FRAMEWORK FOR CONVERGENCE AND CONTINUOUS DEPENDENCE OF ESTIMATES IN INVERSE PROBLEMS FOR DISTRIBUTED PARAMETER SYSTEMS

by

H.T. Banks and K. Ito

March 1987 LCDS/CCS #87-20

A THEORETICAL FRAMEWORK FOR CONVERGENCE AND CONTINUOUS DEPENDENCE OF ESTIMATES IN INVERSE PROBLEMS FOR DISTRIBUTED PARAMETER SYSTEMS

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March 1987

one can show that $A(q)$ generates a C_0 semigroup $T(t;q)$ on H. If $b > 0$ in (B´), this semigroup is uniformly exponentially stable and if in (B´) we replace $|\psi|_\mu$ by $|\psi|_V$ and have $b > 0$, the semigroup is analytic. For Euler-Bernoulli beams, the general case handles viscous and spatial hysteresis damping (with uniform exp. stability if the damping coefficient is strictly positive) while Kelvin-Voigt damping is included in the analytic semigroup case.

Identification problems for these second order systems may be formulated in a manner analogous to the first order case outlined above; a convergence/stability theory under weak compactness assumptions (typically Q can be taken as a subset of $C(\Omega)$ with the supremum metric) can be given using the resolvent form of the Trotter-Kato theorem. This yields results that are a significant improvement over those currently in the research literature. Details will be given in a forthcoming manuscript.

ACKNOWLEDGEMENTS

Research was supported in part under grants NSF MCS-8504316, NASA NAG-1-517, AFOSR-84-0398, and AFOSR-F49620-86-C-0111.

Part of the research was carried out while the authors were visiting scientists at ICASE, NASA Langley Research Center which is operated under NASA Contracts NAS1-17070 and NAS1-18107.

REFERENCES

[B] H. T. Banks, On a variational approach to some parameter estimation problems, Distributed Parameter Systems, Lec. Notes in Control and Inf. Sci., 75 (1985), pp. 1-23, Springer.

[BCK] H. T. Banks, J. M. Crowley, and K. Kunisch, Cubic spline approximations techniques for parameter estimation in distributed systems, IEEE Trans. Auto. Control AC-28 (1983), pp. 773-786.

[BCR] H. T. Banks, J. Crowley, and I. G. Rosen, Methods for the identification of material parameters in distributed models for flexible structures, Mat. Applicada e Comput., 5 (1986).

[BD] H. T. Banks and P. L. Daniel (Lamm), Estimation of variable coefficients in parabolic distributed systems, IEEE Trans. Auto. Control AC-30 (1985), pp. 386-398.

[BI] H. T. Banks and D. W. Iles, On compactness of admissible parameter sets: Convergence and stability in inverse problems for distributed parameter systems, ICASE Rep. 86-38, NASA Langley Research Center, Hampton, VA, June 1986.

A Theoretical Framework for Convergence and Continuous Dependence of Estimates in Inverse Problems for Distributed Parameter Systems

Abstract

In this note we announce a framework in which one can treat very general classes of parameter estimation problems for distributed systems. Using this approach one can obtain both convergence and continuous dependence (stability) results under very weak regularity and compactness assumptions on the set of admissible parameters. *The framework includes both first and second* order abstract systems of importance in applications.

Oecologia (Berlin) (1987) 74:356–362

Oecologia

Parameter estimation techniques for interaction and redistribution models: a predator-prey example

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Summary. The use of parameter estimation techniques for partial differential equations is illustrated using a predator-prey model. Whereas ecologists have often estimated parameters in models, they have not previously been able to do so for models that describe interactions in heterogeneous environments. The techniques we describe for partial differential equations will be generally useful for models of interacting species in spatially complex environments and for models that include the movement of organisms. We demonstrate our methods using field data from a ladybird beetle (*Coccinella septempunctata*) and aphid (*Uroleucon nigrotuberculatum*) interaction. Our parameter estimation algorithms can be employed to identify models that explain better than 80% of the observed variance in aphid and ladybird densities. Such parameter estimation techniques can bridge the gap between detail-rich experimental studies and abstract mathematical models. By relating the particular best-fit models identified from our experimental data to other information on *Coccinella* behavior, we conclude that a term describing local taxis of ladybirds towards prey (aphids in this case) is needed in the model.

Key words: Parameter estimation – Species interaction – redistribution – Heterogeneous environments

Although mathematical models of species interactions command much attention in the ecological literature, their usefulness is sometimes questioned (see, for example, Simberloff 1983). One problem with models in ecology is that they are difficult to explicitly relate to field data. By assuming that interacting populations are at an equilibrium, Schoener (1974) and Belovsky (1984) have been able to use numerical least-squares methods to estimate parameters in models on the basis of field data. These efforts, however, depend on explicit analytic representations for the functions (solutions) being observed or fit to data. Usually such explicit solutions are possible only for the simplest of models (Mueller and Ayala 1981) or for the equilibrium points of more complex models. Furthermore, in the ecological literature parameter estimation techniques have not, to our knowledge, been applied to transient dynamic models of spatially structured interactions (i.e., interactions in which densities vary in space and are altered by dispersal). Since there is a growing interest in non-equilibrium models (Chesson and Case 1985) and in models that allow organisms to move about in a heterogeneous environment (Levin 1974; Wiens et al. 1985), there is a need for parameter estimation techniques that do not rely on writing down a particular formula which represents the solution to a model.

In this paper, we present parameter estimation techniques for models that take the form of partial differential equations. Such models can be used to represent species interactions in heterogenous environments plus a wide variety of movement behaviors. The mathematical theory behind these techniques is developed elsewhere (Banks and Kareiva 1983; Banks et al. 1985; Banks and Murphy 1986; 1987). Our goal here is to make experimental ecologists aware that if they collect spatially and temporally structured field data (i.e., densities at position $x_1, x_2, \ldots, x_m$ at each of the times $t_1, t_2, \ldots, t_n$), there is the possibility of using these data to identify parameters in models of interacting populations with dispersal. We will illustrate the use of such parameter estimation techniques by applying our methods to field data we have collected on a predator-prey interaction between ladybird beetles (*Coccinella septempunctata*) and aphids (*Uroleucon nigrotuberculatum*).

The parameter estimation problem for general interaction and redistribution models

Interaction and redistribution of species can be generally modeled by equations of the form

$$\frac{\partial N_1}{\partial t} = \frac{\partial}{\partial x}\left(D_1(t, x, N_1)\frac{\partial N_1}{\partial x}\right) - \frac{\partial}{\partial x}(V_1(t, x, N_1)\, N_1) + f_1(t, x, N_1, N_2), \tag{1a}$$

$$\frac{\partial N_2}{\partial t} = \frac{\partial}{\partial x}\left(D_2(t, x, N_2)\frac{\partial N_2}{\partial x}\right) - \frac{\partial}{\partial x}(V_2(t, x, N_2)\, N_2) + f_2(t, x, N_1, N_2) \tag{1b}$$

where N_1 and N_2 are the densities of two interacting species that vary in space (i.e., with respect to x). The f_1 and f_2 represent the local "kinetics" or "interaction terms" for each species – these could be competition functions, predator-prey terms, or host-disease terms. The first two terms on the right side of (1a) and (1b) describe the movement (or "redistribution") of N_1 and N_2 with respect to x. The movement terms with D_i in the parentheses correspond to

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eters for: nonlinear interaction terms, nonlinear diffusion terms, nonlinear convection terms, and taxis towards or away from other organisms. Thus we will clearly have the ability to identify biologically realistic models of species interactions in spatially heterogeneous environments.

Our approach using parameter estimation is not, however, a panacea for population modelers. Our experience indicates that the models which describe field data well usually include numerous parameters and are difficult to analyze (Banks and Kareiva 1983; Banks et al. 1985). Sometimes we are left with detailed models that can only be explored by exhaustive numerical studies. In addition, there is the danger that so-called "best-fit" models are biological nonsense. The techniques we have used are so powerful that rarely do we fail to find some model which generates numerical solutions closely matched to the data.

Parameter and model identification approaches are best viewed as *one tool* in the arsenal of population biologists. Once population data have led to a suite of models such as examples A–D in Table 1, experiments need to be performed to test the models or components of the models. For instance, using the equations presented in Table 1 we could make predictions about ladybird and aphid population dynamics for any set of initial conditions, and then test those predictions. Alternatively, theoreticians, more interested in general features of models, could examine the properties of models that are simplified, but qualitatively similar to those identified in Table 1.

Ideally, we would like to verify models by direct observation of individual demography and individual movement behavior. Unfortunately, these sorts of observations are not always possible. Indeed, we were able to make these observations on *Coccinella* only when we arranged a 0.1 m × 10 m line of goldenrod as our experimental arena. As the vegetation becomes more two-dimensional (even a one-meter wide strip), detailed behavioral observations become increasingly difficult. In a wide variety of settings (e.g. pest management, studies of the spread of invading organisms, etc.), data on population densities at different times and places may be the only information that is practically available. In these cases, parameter estimation algorithms can help generate reasonable first-guesses for models of population interaction, which can then focus subsequent empirical efforts.

Acknowledgments. H.T. Banks and K.A. Murphy were supported by the National Science Foundation under NSF Grant MCS-8504316 and that Air Force Office of Scientific Research under Contract No. AFOSR-84-0398. Part of their research was carried out while they were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA which is operated under NASA Contract NAS1-17070. P. Kareiva was supported by NSF under grant BSR 8517183. We thank Joy Bergelson for reading an earlier version of this manuscript and offering helpful suggestions.

References

Banks HT (1981) Parameter identification techniques for physiological control systems. In: Lectures in Applied Math. Hoppensteadt F (ed) Am Math Soc, Providence, RI, 19:361–383

Banks HT (1985) On a variational approach to some parameter estimation problems. In: Distributed Parameter Systems. Springer Lec. Notes in Control & Info Sci. 75:1–23

Banks HT, Kareiva P (1983) Parameter estimation techniques for transport equations with applications to population dispersal and tissue bulk flow models. J Math Biol 17:253–273

Banks HT, Murphy KA (1986) Quantitative modeling of growth and dispersal in population models. LCDS Report no 86-4, Brown University, and Proc International Sympl on Math Biol, Kyoto, Nov. 10–15, 1985, Springer LN in Biomath 71:98–109

Banks HT, Murphy KA (1987) Estimation of nonlinearities in parabolic models for growth, predation and dispersal of populations. LCDS/CCS Report No. 87-36, Brown University

Banks HT, Crowley JM, Kunisch K (1983) Cubic spline approximation techniques for parameter estimation in distributed systems. IEEE Trans Auto Control AC-28:773–786

Banks HT, Kareiva PM, Lamm PK (1985) Modeling insect dispersal and estimating parameters when mark-release techniques may cause initial disturbances. J Math Biol 22:259–277

Belovsky G (1984) Moose and snowshoe hare competition and a mechanistic explanation of foraging theory. Oecologia (Berlin) 61:150–159

Chesson PL, Case TJ (1985) Nonequilibrium community theories: chance, variability, history and coexistence. In: Community Ecology. Diamond J, Case T (eds) Harper & Row, NY, pp 229–239

Kareiva P (1985) Patchiness, dispersal, and species interactions: consequences for communities of herbivorous insects. In: Community Ecology. Diamond J, Case T (eds) Harper and Row, NY, pp 192–206

Kareiva P (1986) Trivial movement and foraging by crop colonizers. In: Ecological Theory and Pest Management. Kogan M (ed) Wiley-Interscience, NY, pp 99–122

Kareiva P, Odell G (1987) Predator aggregation and the containment of prey outbreaks as a result of area restricted search behavior. Am Nat (in press)

Levin S (1974) Dispersion and population interactions. Am Nat 108:207–228

Mueller LD, Ayala FJ (1981) Dynamics of single-species population growth: experimental and statistical analysis. Theor Pop Biol 20:101–117

Obrycki J, Nechols J, Tauber M (1982) Establishment of a European ladybeetle in New York State. New York Food and Life Sciences Bulletin 94:1–4

Schoener TW (1974) Competition and the form of habitat shift. Theor. Pop. Biol 6:265–307

Schultz MH (1973) Spline Analysis. Prentice Hall, Englewood Cliffs

Simberloff D (1983) Competition theory, hypothesis testing, and other community ecological buzzwords. Am Nat 122:626–635

Wiens JH, Addicott JF, Case TJ, Diamond J (1985) The importance of spatial and temporal scale in ecological investigations. In: Community Ecology. Diamond J, Case T (eds) Harper & Row, NY, pp 145–153

Received May 30, 1987

ESTIMATION OF NONLINEARITIES IN PARABOLIC MODELS FOR GROWTH, PREDATION AND DISPERSAL OF POPULATIONS

by

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August 1987 LCDS/CCS #87-36

ESTIMATION OF NONLINEARITIES IN PARABOLIC MODELS FOR GROWTH, PREDATION AND DISPERSAL OF POPULATIONS

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* This research was supported in part under grants NSF MCS-8504316, AFOSR-84-0398 and contract AFOSR-F49620-86-C-0111.

+ This research was supported in part under grant AFOSR-86-0256.

Part of this research was carried out while the authors were visiting scientists at ICASE, NASA Langley Research Center, Hampton,VA, which is operated under NASA contracts NAS1-17070 and NAS1-18107.

ABSTRACT

A convergence theory is given for approximation techniques to treat inverse problems involving systems of nonlinear parabolic partial differential equations. These techniques can be used to estimate density-dependent dispersal coefficients in population models, as well as nonlinear growth and predation terms. Numerical experiences with the resulting algorithms on both conventional (scalar) and vector computers are reported along with an indication of performance of the methods with field data from prey-predator experiments.

Proceedings of IMACS/IFAC Symposium on
Modelling and Simulation of
Distributed Parameter Systems
Hiroshima, October 6-9, 1987

COMPUTATIONAL TECHNIQUES FOR ESTIMATION AND CONTROL OF DISTRIBUTED PARAMETER SYSTEMS

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We survey some problems and related recent results in computational methods for two classes of problems for distributed parameter systems: parameter estimation or inverse problems and feedback control.

I. INTRODUCTION

In this lecture we shall give a survey of some of our recent and current efforts in the Center for Control Sciences at Brown University. We shall restrict our attention to research projects involving theoretically sound computational aspects of parameter estimation and feedback control in problems described by partial differential equations. As time permits, the topics listed below will be discussed.

II. THEORETICAL FRAMEWORK FOR PARAMETER ESTIMATION

We consider first order systems dependent on parameters $q \in Q$ as given by an abstract equation

$$\begin{aligned} \dot{u}(t) &= A(q)u(t)+F(t,q) \\ u(0) &= u_0(q) \end{aligned} \tag{2.1}$$

in a Hilbert space H. We seek to estimate the parameters q using observations $\hat{u}_i$ for $u(t_i;q)$ in a least-squares fit criterion. We assume that Q is a metric space with metric d and for $q \in Q$, $A(q)$ is the infinitesimal generator of a C_0 semigroup on H. We assume that $A(q)$ is defined through a parameter dependent sesquilinear form $\sigma(q)(\cdot,\cdot)$ associated with the weak or variational form of (2.1). Briefly, let V be a Hilbert space that is continuously and densely imbedded in H. Denote a family of parameter dependent sesquilinear forms by $\sigma(q): V \times V \to \mathbb{C}, q \in Q$. We assume that σ possesses the following properties:

(A) Continuity: For $q, \tilde{q} \in Q$, we have for all $\phi, \psi \in V$

$$|\sigma(q)(\phi,\psi) - \sigma(\tilde{q})(\phi,\psi)| \le d(q,\tilde{q})|\phi|_V |\psi|_V.$$

(B) Coercivity: There exists $c_1 > 0$ and some λ such that for $q \in Q, \phi \in V$ we have

$$\sigma(q)(\phi,\phi) + \lambda|\phi|_H^2 \ge c_1|\phi|_V^2.$$

(C) Boundedness: There exist $c_2 > 0$ such that for $q \in Q$, $\phi, \psi \in V$ we have

$$|\sigma(q)(\phi,\psi)| \le c_2|\phi|_V|\psi|_V.$$

Under these assumptions, σ defines in the usual manner operators $A(q)$ such that $\sigma(q)(\phi,\psi) = \langle -A(q)\phi,\psi\rangle_H$ for $\phi \in \operatorname{dom}(A(q))$, $\psi \in V$ with $\operatorname{dom}(A(q))$ dense in V. Furthermore, $A(q)$ is the generator of an analytic semigroup $T(t;q)$ on H (indeed, $A(q)$ is sectorial with $(\lambda I - A(q))\operatorname{dom}(A(q)) = H$). Property (B) guarantees that the resolvent operator $R_\lambda(A(q)) \equiv (\lambda I - A(q))^{-1}$ exists as a bounded operator on H.

We may then consider Galerkin type approximations in the context of these sesquilinear forms. Let H^N be a family of finite dimensional subspaces of H satisfying $P^N z \to z$ for $z \in H$ where P^N is the orthogonal projection of H onto H^N. We further assume that $H^N \subset V$ and possesses certain V-approximation properties. If we now consider the restriction of $\sigma(q)(\cdot,\cdot)$ to H^N, we obtain operators $A^N(q): H^N \to H^N$ which, because of (B), satisfy a uniform dissipative inequality and can be shown to generate semigroups $T^N(t;q)$ in H^N.

Using a resolvent convergence form of the Trotter-Kato theorem to establish convergence properties of $T^N(t;q^N)$ to $T(t;q)$ whenever $\{q^N\}$ is any arbitrary sequence converging to q in d-metric, one can establish a convergence and stability theory for the resulting approximating least-squares estimation problems.

Among the examples that can be treated immediately with the above theory are the usual coefficient and boundary parameter identification problems for parabolic systems (in this case $H = L_2(\Omega)$ and $V = H_0^1(\Omega)$ or some other appropriately chosen subspace of $H^1(\Omega)$ with modified boundary conditions). This theoretical framework can also readily be used for domain identification problems arising in thermal tomography.

The first step of our hybrid method requires the solution of n(m+p) simultaneous equations, while each iteration of the usual Newton-Kleinman step requires the solution of a Lyapunov equation for the nxn symmetric estimates of $\tilde{P}$.

However, one can use factorization ideas and the Smith method for Lyapunov equations to reformulate the Newton-Kleinman method as a direct iterative method for the mxn gain K, thereby providing additional computational advantages. To speed up our calculations and improve convergence in the Smith algorithm, we have used a variable stepsize Smith method to solve the Lyapunov equations.

Our initial numerical experiments with this hybrid algorithm have provided quite encouraging results. For example, if n is the dimension of our approximating systems, we have observed in our examples with 1-dimensional parabolic systems that the traditional eigenvector based methods (Potter, Laub-Schur) exhibit computational effort (and time) that grows like n^3 whereas the hybrid method time grows like n.

IV. CONCLUDING REMARKS

Much of the research discussed in this survey lecture involves very recent and current results. Some of it involves joint efforts with other investigators (including K. Ito, Y. Wang, F. Kojima, D.J. Inman, H. Cudney, R. Fabiano, G. Rosen, S. Reich and C. Wang) and in several cases manuscripts with the detailed results are currently being written. At the time of the lecture we shall provide a bibliography available on these and related results.

V. ACKNOWLEDGEMENTS

This research was supported in part under grants NSF MCS-8504316, NASA NAG-1-517, AFOSR-84-0398, and AFOSR-F49620-86-C-0111. Part of the research was carried out while the author was a visiting scientist at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center which is operated under NASA Contracts NAS1-17070 and NAS1-18107.

Proceedings of the 26th Conference
on Decision and Control
Los Angeles, CA • December 1987

PARAMETER IDENTIFICATION TECHNIQUES FOR THE ESTIMATION OF DAMPING IN FLEXIBLE STRUCTURE EXPERIMENTS

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Abstract

We report on our use of spline based inverse procedures to estimate damping coefficients in distributed parameter systems for flexible structures. Damping models involving viscous (air) damping and Kelvin-Voigt damping in an Euler-Bernoulli framework are used to analyze data from vibration experiments with composite material beams.

I. Introduction

In this presentation we report on continuing efforts that we have pursued for the past year. These efforts involve a combination of experimental investigations with the simultaneous development of the mathematical and computational aspects of a theoretical methodology to support the experiments.

Our long term goals include the understanding of damping mechanisms in complex distributed structures constructed from composite materials. Our quest has obvious motivation from and potential consequences for the design of control (active and passive) systems for large flexible space structures.

The initial efforts on which we report here entail the testing and development of spline based approximation techniques for inverse problems arising in attempts to model and quantify damping in composite material beams. Substantial previous mathematical efforts along with numerical tests on simulated data can be found in the literature (see [BCR], [BR] for a sample of some of these results and references to other work).

We briefly outline the underlying mathematical ideas, describe the experimental equipment employed, and then report on two representative experiments and our findings using our techniques with the data from these experiments.

II. Mathematical Foundations

The basic mathematical model that we have considered in connection with the efforts on which we report in this note consists of the equations for a cantilevered Euler-Bernoulli equation with tip mass, damping, and an applied transverse force. Specifically we take as our model the system for transverse vibrations of a long slender (length ℓ) cantilevered beam:

$$\rho \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} + c_D I \frac{\partial^5 u}{\partial x^4 \partial t} + \gamma \frac{\partial u}{\partial t} = f(t, x_p, \tau), \quad 0 < x < \ell,\ t > 0, \tag{2.1}$$

$$u(t,0) = \frac{\partial u}{\partial x}(t,0) = 0, \ t > 0 \tag{2.2}$$

$$EI \frac{\partial^2 u}{\partial x^2}(t,\ell) + c_D I \frac{\partial^3 u}{\partial x^2 \partial t}(t,\ell) = 0, \ t > 0, \tag{2.3}$$

$$EI \frac{\partial^3 u}{\partial x^3}(t,\ell) + c_D I \frac{\partial^4 u}{\partial x^3 \partial t}(t,\ell) = m \frac{\partial^2 u}{\partial t^2}(t,\ell), \ t > 0, \tag{2.4}$$

$$u(0,x) = \phi(x), \ \frac{\partial u}{\partial t}(0,x) = \psi(x), \ 0 < x < \ell. \tag{2.5}$$

Here ρ is the linear mass density, m is the tip mass, $c_D I$ and γ are damping coefficients for Kelvin-Voigt and viscous (air) damping, respectively, and f is a parameter dependent (x_p for the point of impact, τ for the duration) term representing a transverse force such as that obtained with a force hammer. The functions ϕ and ψ represent given initial displacement and velocity of the beam.

We assume that we are given observations - in this case, of tip acceleration - from which we wish to estimate unknown parameters. In general these include the stiffness coefficient EI as well as the damping parameters $c_D I$, γ and the input parameter τ. Denoting this collection of unknown parameters by $q = (EI, c_D I, \gamma, \tau)$, we may formulate this inverse problem as the least-squares estimation problem of minimizing the functional

$$J(q) = \sum_i \left| \frac{\partial^2 u}{\partial t^2}(t_i, \ell; q) - a(t_i) \right|^2 \tag{2.6}$$

over an admissible class Q of parameter values. In this fit-to-data criterion $\{a(t_i)\}$ are the observations (data) for the tip acceleration (at $x = \ell$) of the beam.

This estimation problem cannot, of course, be solved analytically; however, one can employ iterative optimization schemes coupled with an approximation method for the infinite dimensional system (2.1) - (2.5). For the efforts discussed here, we approximated the system via Galerkin procedures using cubic B-spline elements as explained in [BGRW]. This results in an approximate solution u^N which satisfies a readily-solved finite dimensional system approximating (2.1) - (2.5). We use this in the criterion (2.6) in place of u, thereby obtaining a family of approximating estimation problems involving minimization over Q of

$$J^N(q) = \sum_i \left| \frac{\partial^2 u^N}{\partial t^2}(t_i, \ell; q) - a(t_i) \right|^2. \tag{2.7}$$

Solving these, we obtain a sequence of estimates $\{\bar{q}^N\}$ for best-fit parameters in the original estimation problem. A theory for convergence of these estimates can be obtained (see e.g. [BCR],[BR],[BGRW] for details), but we shall not discuss this aspect of the computational techniques here since the focus of this report is the <u>use</u> of these techniques in studying damping mechanisms in composite material structures.

We have assumed here that the observations are taken with tip accelerometers since that is precisely the case for the experiments discussed below. However, we could equally well assume velocity observations are taken; indeed, we have also used these methods with experiments in which a laser vibrometer was used to obtain velocity measurements. These and related findings will be detailed in a separate manuscript currently in preparation.

Furthermore, in our findings discussed here, we have concentrated on attempts to characterize the damping in terms of Kelvin-Voigt and/or viscous (air) damping. There is considerable evidence to suggest that damping mechanisms in composite materials are significantly more complex than the ones described by a combination of viscous and Kelvin-Voigt damping. Indeed, current conjectures include models that involve hysteretic or hereditary (time and/or space) effects. Recent theoretical results [BI] provide a convenient mathematical framework for treating inverse problems for such models; we are currently using the associated computational techniques to investigate the possibility of describing experimental data in terms of models with hysteretic damping.

III. Experimental Procedures

To test the above described estimation procedures and initiate our damping mechanism studies a series of experiments were carried out at the Mechanical Systems Laboratory located at SUNYAB. Cantilevered composite beams with a removable tip mass were used as the test structures. Each beam consisted of a one meter long piece

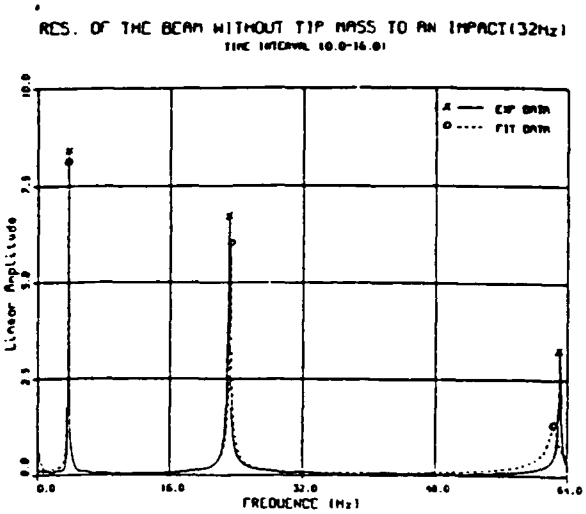

Figure 6

If we try to estimate γ and c_DI using modal techniques (via % critical damping and principles relating ζ to γ and c_DI similar to that noted in the discussion of the previous experiment), we obtain values γ_{cal} = .2982 slug/(ft sec) and c_DI_{cal} = .4511 slug ft^3/sec. If we use these in the model equations (2.1) - (2.5), we find that the response is damped out much too quickly and does not resemble the experimental data.

A final comment on our findings in this example has important implications for use of modal methods in attempts to understand damping mechanisms in distributed parameter structures. While we found that the viscous damping (γ) was extremely important in the overall modeling of the low frequency mode and the Kelvin-Voigt damping (c_DI) was most important in modeling correctly the two high frequency modes, the damping does not decouple on the modes. We belive this poses serious difficulties for any attempts to understand damping mechanisms using only modal techniques.

V. Acknowledgements

This research was supported in part under grants NASA NAG-1-517 (H.T.B. and Y.W.), AFOSR-84-0398 (H.T.B.), AFOSR-F49620-86-C-0111 (H.T.B., Y.W., D.J.I.), AFOSR-85-0220 (D.J.I.), NSF-MSM-8351807 (D.J.I.), NASA Training Grant NGT 33183802 (H.C.) and DOD Equipment Grant AFOSR-85-0119 (D.J.I.).

References

[BCR] Banks, H.T., Crowley, J.M., and Rosen, I.G., Methods for the identification of material parameters in distributed models for flexible structures, Mat. Aplic. Comp. 5 (1986), 139-168.

[BGRW] Banks, H.T., Gates, S.S., Rosen, I.G., and Wang, Y., The identification of a distributed parameter model for a flexible structure, ICASE Rep. No. 86-71, October, 1986, NASA Langley Res. Ctr., Hampton; SIAM J. Control Opt., to appear.

[BI] Banks, H.T., and Ito, K., A theoretical framework for convergence and continuous dependence of estimates in inverse problems for distributed parameter systems, LCDS/CCS Rep. 87-20, March, 1987, Brown University; Applied Math. Lett. Vol 0, No 1, June, (1987) 31-35.

[BR] Banks, H.T., and Rosen, I.G., Computational methods for the identification of spatially varying stiffness and damping in beams, *Control-Theory and Adv. Tech.* 3 (1987), 1-32.

[SDRC] SDRC 1985 User Manual for Modal Analysis 9.0, General Electric CAE International, Milford, Ohio.

[WM] Wilson, M.L. and Miserentino, R., 1986 "Pultrusion Process Development for Long Space Boom Models", Proceedings of the 41st Annual Conference of the Reinforced Plastics/Composites Institute, Society of the Plastics Industry, Paper No. 6-D.

AN APPROXIMATION THEORY FOR THE IDENTIFICATION OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS

by

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April 1988 LCDS/CCS #88-8

AN APPROXIMATION THEORY FOR THE IDENTIFICATION OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS+

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*This research was supported in part under grants NSF MCS-8504316, NASA NAG-1-517, AFOSR-84-0398, and AFOSR-F49620-86-C-0111.

**This research was supported in part by the Fund for the Promotion of Research at The Technion and by the Technion VPR Fund.

***This research was supported in part under grants AFOSR-84-0393 and AFOSR-87-0356.

+Part of this research was carried out while the first and third authors were visiting scientists at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA, which is operated under NASA contract NAS1-18107.

An Approximation Theory for the Identification of Nonlinear Distributed Parameter Systems

ABSTRACT

An abstract approximation framework for the identification of nonlinear distributed parameter systems is developed. Inverse problems for nonlinear systems governed by strongly maximal monotone operators (satisfying a mild continuous dependence condition with respect to the unknown parameters to be identified) are treated. Convergence of Galerkin approximations and the corresponding solutions of finite dimensional approximating identification problems to a solution of the original infinite dimensional identification problem is demonstrated using the theory of nonlinear evolution systems and a nonlinear analog of the Trotter-Kato approximation result for semigroups of bounded linear operators. The nonlinear theory developed here is shown to subsume an existing linear theory as a special case. It is also shown to be applicable to a broad class of nonlinear elliptic operators and the corresponding nonlinear parabolic partial differential equations to which they lead. An application of the theory to a quasilinear model for heat conduction or mass transfer is discussed.

SEMIGROUP THEORY AND

NUMERICAL APPROXIMATION FOR

EQUATIONS IN LINEAR VISCOELASTICITY

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May 1988

* This research was supported in part by the Air Force Office of Scientific Research under grant AFOSR-84-0398 and contract AFOSR-F49620-86-C-011, by the National Science Foundation under grant MCS-8504316, and the National Aeronautics and Space Administration under NASA Grant NAG-1-517.

Semigroup Theory and Numerical Approximation for Equations in Linear Viscoelasticity

Abstract

We consider the following abstract integro-differential equation

$$\ddot{u}(t) + A\left[Eu(t) - \int_{-r}^{0} g(s)u(t+s)\,ds\right] = f(t)$$

on a Hilbert space. Such equations arise in the modeling of linear viscoelastic beams. The equation is reformulated as an abstract Cauchy problem, and several approximation schemes are discussed. Well-posedness and convergence results are given in the context of linear semigroup theory. Results of numerical eigenvalue calculations for various approximation schemes are discussed.

The Augmented Lagrangian Method for Equality and Inequality Constraints in Hilbert Spaces

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April 1986

* This research was supported by the Air Force Office of Scientific Research under Grant AFOSR-84-0398 and AFOSR-85-0303, and the National Aeronautics and Space Administration under Grant NAG-1-517.

** This research supported in part by the Funds for Förderıng der wissenschafftichen Forsching under S3206 and AFOSR-84-0398. Part of this work was performed while the author was visiting the Division of Applied Mathematics, Brown University.

Augmented Lagrangian Method for Equality and Inequality Constraints in Hilbert Spaces

Abstract

In this paper, we consider a class of Lagrange multiplier methods, called the augmented Lagrangian method for the minimizations with equality and inequality constraints in Hilbert spaces. We obtain a local square-root convergence of this method without strict complementary conditions when finite many inequality constraints are augmented. This result seems to even generalize the known results in the finite dimensional case.

On the Regularity of Solutions of an Operator Riccati Equation Arising in Linear Quadratic Optimal Control Problems for Herediatry Differential Systems

by

Kazufumi Ito

May 1986 LCDS #86-27

ON THE REGULARITY OF SOLUTIONS OF AN OPERATOR RICCATI EQUATION ARISING IN LINEAR QUADRATIC OPTIMAL CONTROL PROBLEMS FOR HEREDITARY DIFFERENTIAL SYSTEMS*

by

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* This research was supported by the Air Force Office of Scientific Research under grants AFOSR-84-0398 and AFOSR-85-0303, and the National Aeronautics and Space Administration under grant NAG-1-517.

On the Regularity of Solutions of an Operator Riccati Equation Arising in Linear Quadratic Optimal Control Problems for Hereditary Differential Systems

Abstract

In this paper operator Riccati equations of evolution and algebriac types arising in the linear quadratic optimal control problem for hereditary differential systems are considered. Regularity results of their solutions are obtained. Spectrally, for the system with single point delay we show that the optimal feedback kernel function Π^{10} is C^{∞}. This study is motivated by our numerical experiments [8] in computing the optimal feedback gain via an algebraic Riccati equation, which indicate that such a function is smooth. The regularity result is essential in obtaining a rate of convergence of numerical approximations of the optimal feedback gain operator.

APPROXIMATION OF INFINITE DELAY AND VOLTERRA TYPE EQUATIONS

by

K. Ito and F. Kappel

September 1986 LCDS #86-35

APPROXIMATION OF INFINITE DELAY AND VOLTERRA TYPE EQUATIONS

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1) Supported in part by the Air Force Office of Scientific Research under Contracts AFOSR-84-0398 and AFOSR-85-0303 and the National Aeronautics and Space Administration under NASA Grant NAG-1-517.

2) Supported in part by the Air Force Office of Scientific Research under Contract AFOSR-84-0398 and in part by the Fonds zur Förderung der wissenschaftlichen Forschung, Austria, under project No. S3206.

"Approximation of infinite delay and Volterra type equations"

by K. Ito and F. Kappel

S u m m a r y

Linear autonomous functional differential equations of neutral type include Volterra integral and integrodifferential equations as special cases. The paper considers numerical approximation of solutions to these equations by first converting the initial value problem to an abstract Cauchy problem in a product space ($\mathbb{R}^n \times$ weighted L^2-space) and then using abstract approximation results for C_0-semigroups combined with Galerkin type ideas. In order to obtain concrete schemes subspaces of Legendre and Laguerre polynomials are used. The convergence properties of the algorithms are demonstrated by several examples.

Running head: Volterra type equations

Subject classification: 34G10, 34K99, 45D05, 45J05, 45L10

FINITE DIMENSIONAL COMPENSATORS FOR INFINITE DIMENSIONAL SYSTEMS VIA GALERKIN-TYPE APPROXIMATION

by

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Finite Dimensional Compensators for Infinite Dimensional Systems via Galerkin-Type Approximation

Abstract

In this paper we discuss existence and construction of stabilizing compensators for linear time-invariant system defined on Hilbert spaces. We establish an existence result using Galerkin-type approximation in which independent basis elements are used instead of the complete set of eigenvectors. A design procedure based on approximate solutions of optimal regulator and optimal observer via Galerkin-type approximation is given and the Schumacher approach is used to reduce a dimension of compensators. A detail discussion for parabolic and hereditary differential systems is included.

THE AUGUMENTED LANGRANGIAN METHOD FOR PARAMETER ESTIMATION IN ELLIPTIC SYSTEMS

by

Kasufumi Ito and Karl Kunisch

April 1987 LCDS/CCS #87-37

The Augmented Langrangian Method for Parameter Estimation in Elliptic Systems

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October. 1987

* This research was supported in part by the Air Force Office of Scientific Research under grants AFOSR-84-0398 and AFOSR-85-0303, and the National Aeronautics and Space Administration under grant NAG-1-1517, and by NSF under Grant No. UINT-8521208

** This reasearch was supported in part by the Fonds for Förderung der wissenschaftlichen Forschung under S3206 and P6005 and by AFOSR-84-0398. Part of this work was performed while the author was visiting the Division of Applied Mathematics, Brown University.

Abstract

In this paper a new technique for the estimation of parameters in elliptic partial differential equations is developed. It is a hybrid method combining the output - least - square and the equation error method. The new method is realized by an augmented Lagrangian formulation and convergence as well as rate of convergence proofs are provided. Technically the critical step is the verification of a coercivity estimate of an appropriately defined Lagrangian functional. To obtain this coercivity estimate a seminorm regularization technique is used.

CHANDRASEKAR EQUATIONS FOR INFINITE DIMENSIONAL SYSTEMS: PART II. UNBOUNDED INPUT AND OUTPUT CASE

by

Kazufumi Ito and Robert K. Powers

April 1987 LCDS/CCS #87-16

Chandrasekar Equations for Infinite Dimensional Systems:

Part II. Unbounded Input and Output Case

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Chandrasekar Equations for Infinite Dimensional Systems: Part II. Unbounded Input and Output Case

Abstract

A set of equations known as Chandrasekhar equations arising in the linear quadratic optimal control problem is considered. In this paper, we consider the linear time-invariant systems defined in Hilbert spaces involving unbounded input and output operators. For a general class of such systems, we derive the Chandrasekhar equations and establish the existence, uniqueness, and regularity results of their solutions.

STRONG CONVERGENCE AND CONVERGENCE RATES OF APPROXIMATING SOLUTIONS FOR ALGEBRAIC RICCATI EQUATIONS IN HILBERT SPACES

by

Kazufumi Ito

April 1987 LCDS/CCS #87-15

STRONG CONVERGENCE AND CONVERGENCE RATES OF APPROXIMATING SOLUTIONS FOR ALGEBRAIC RICCATI EQUATIONS IN HILBERT SPACES*

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*Invited lecture, Third International Conference on Control and Identification of Distributed Parameter Systems, July 6-12, 1986, Vorau, Austria.

Strong Convergence and Convergence Rates of Approximating Solutions for Algebraic Riccati Equations in Hilbert Spaces

Abstract

In this paper, we consider the linear quadratic optimal control problem on infinite time interval for linear time-invariant systems defined on Hilbert spaces. The optimal control is given by a feedback form in terms of solution Π to the associated algebraic Riccati equation (ARE). A Ritz type approximation is used to obtain a sequence Π^N of finite dimensional approximations of the solution to ARE. A sufficient condition that shows Π^N converges strongly to Π is obtained. Under this condition, we derive a formula which can be used to obtain a rate of convergence of Π^N to Π. We demonstrate and apply the results for the Galerkin approximation for parabolic systems and the averaging approximation for hereditary differential systems.

SIAM J. CONTROL AND OPTIMIZATION
Vol. 25, No. 3, May 1987

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CHANDRASEKHAR EQUATIONS FOR INFINITE DIMENSIONAL SYSTEMS*

KAZUFUMI ITO† AND ROBERT K. POWERS‡

Abstract. In this paper we derive the Chandrasekhar equations for linear time invariant systems defined on Hilbert spaces using a functional analytic technique. An important consequence of this is that the solution to the evolutional Riccati equation is strongly differentiable in time and one can define a "strong" solution of the Riccati differential equation. A detailed discussion on the linear quadratic optimal problem for hereditary differential systems is also included.

Key words. Chandrasekhar equations, Riccati operator, regularity results, infinite dimensional systems

AMS(MOS) subject classification. 49

1. Introduction. The Chandrasekhar equations [14] are an alternative form to the Riccati equations from which the optimal feedback gain operator may be calculated directly. If the system has a small number of inputs and outputs, the Chandrasekhar algorithm offers significant reduction in the computational complexity for determining the optimal feedback gain. As observed in [20], this is much more evident in the infinite dimensional case if the optimal feedback gain operator is calculated numerically using some approximation method. In this case, the number of states grows linearly to the order of approximation.

The purpose of this paper is to derive Chandrasekhar equations for systems defined by evolution equations on Hilbert spaces in which the input and output operators are assumed to be bounded. The form of the Chandrasekhar equations derived immediately implies that the solution of the associated Riccati equation is strongly differentiable in time, and it allows us to define a "strong" solution of the Riccati equation. Another important consequence of this is that the optimal control for the linear quadratic regulator (LQR) problem is continuously differentiable if the initial datum is sufficiently smooth.

The Chandrasekhar equations for infinite dimensional systems have been discussed in [4] and [7] using a Lions-type framework [17]. However, the equations derived in [4] and [7] are satisfied in the distributional sense. In [22], Sorine derived a set of Chandrasekhar equations satisfied in a strong sense for parabolic systems. Sorine's derivation relied on the analyticity of the semigroup and thus does not apply to general systems. Our approach differs from those above in that it uses an approximation technique. A sequence of approximating optimal control problems is chosen for which the Chandrasekhar equations may be derived as in the finite dimensional case (see [6], [14], and [16]). Convergence is then established and the appropriate equations are shown to be satisfied. In this paper, our considerations are restricted to the LQR problem, but the results are also applicable to the Kalman filtering problem [8].

* Received by the editors January 22, 1985; accepted for publication (in revised form) March 11, 1986. This research was supported by the National Aeronautics and Space Administration under NASA contract NAS1-17070 while the authors were in residence at the Institute for Computer Application in Science and Engineering, NASA Langley Research Center, Hampton, Virginia 23665.

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A UNIFORMLY DIFFERENTIABLE APPROXIMATION SCHEME
FOR DELAY SYSTEMS USING SPLINES

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JUNE 1987 No. 94-1987

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[1)] Work done by this author was supported by AFOSR under Contract No. F-49620-86-C-011, by NASA under grant No. NAG-1-517 and by NSF under Grant No. UINT-8521208.

[2)] Work done by this author was supported by AFOSR under Grant No. 84-0398 and by FWF (Austria) under Grants Nos. S3206 and P6005

A UNIFORMLY DIFFERENTIABLE APPROXIMATION SCHEME FOR DELAY SYSTEMS USING SPLINES

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Abstract

A new spline-based scheme is developed for linear retarded functional differential equations within the framework of the semigroups on the Hilbert space $R^n \times L^2$. The approximating semigroups preserve (uniformly) the logarithmic sectorial property (i.e., the differentiability) of the solution semigroup. We prove the convergence of the scheme both in $W^{1,2}$, $R^n \times L^2$, and using the uniform differentiability we are able to establish error estimates and obtain the quadratic convergence for a class of initial data. We also apply the scheme for computing the feedback solutions to the linear quadratic regulator problem. The uniform differentiability is again the essential basis for obtaining a complete theory for convergence of the Riccati solutions.

1. Introduction and preliminaries

In this paper we consider the linear hereditary control system

$$\dot{x}(t) = \sum_{i=0}^{l} A_i x(t+\theta_i) + \int_{-r}^{0} A(\theta)x(t+\theta)d\theta + Bu(t),\ t \geq 0,$$

$$x(0) = \eta,\ x(\theta) = \phi(\theta) \text{ a.e. on } [-r,0), \tag{1.1}$$

$$y(t) = Cx(t),\ t \geq 0,$$

where $-r = \theta_l < \cdots < \theta_0 = 0$, $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$. Furthermore, $A(\cdot)$ is an $n\times n$-matrix valued square integrable function on $[-r,0]$. It is well-known that, for $(\eta,\phi) \in Z = R^n \times L^2(-r,0;R^n)$ and $u \in L^2_{loc}(0,\infty;R^m)$, system (1.1) admits a unique solution $x \in L^2(-r,T;R^n) \cap H^1(0,T;R^n)$ for any $T > 0$. We define the operators $S(t)$, $t \geq 0$, by

$$S(t)(\eta,\phi) = (x(t),x_t),\ t \geq 0,\ (\eta,\phi) \in Z,$$

where $x(t)$ is the solution of (1.1) with initial data (η,ϕ) and $u(t) \equiv 0$. x_t is the function defined by $x_t(\theta) = x(t+\theta)$, $-r \leq \theta \leq 0$. $S(\cdot)$ is a C_0-semigroup on Z with inifinitesimal generator $\mathcal{A}$ given by (see [1], for instance)

$$\text{dom } \mathcal{A} = \{(\eta,\phi) \in Z \mid \phi \in H^1(-r,0;R^n),\ \eta = \phi(0)\},$$

$$\mathcal{A}(\phi(0),\phi) = (L(\phi),\dot{\phi}) \text{ for } (\phi(0),\phi) \in \text{dom } \mathcal{A},$$

where

$$L\phi = \sum_{i=0}^{l} A_i\phi(\theta_i) + \int_{-r}^{0} A(\theta)\phi(\theta)d\theta,\ \phi \in C(-r,0;R^n).$$

If we define the input operator $\mathcal{B}$: $R^m \to Z$ and the output operator C: $Z \to R^p$ by

$$\mathcal{B}u = (Bu,0),\ u \in R^m,$$

$$\mathcal{C}(\eta,\phi) = C\eta,\ (\eta,\phi) \in Z, \tag{1.2}$$

then (1.1) is equivalent to the following abstract system in Z:

$$\dot{z}(t) = \mathcal{A}z(t) + \mathcal{B}u(t),\ t \geq 0,$$

$$z(0) = (\eta,\phi), \tag{1.3}$$

$$y(t) = \mathcal{C}z(t),\ t \geq 0,$$

i.e. a function x: $[0,\infty) \to R^n$ is a solution of (1.1) if and only if the function $z(t) = (x(t),x_t)$, $t \geq 0$, is a mild solution of (1.3) which means

$$z(t) = S(t)(\eta,\phi) + \int_0^t S(t-s)\mathcal{B}u(s)ds,\ t \geq 0. \tag{1.4}$$

The spectrum of $\mathcal{A}$ is only point spectrum and $\lambda \in \sigma(\mathcal{A})$ if and only if $\det \Delta(\lambda) = 0$, where

$$\Delta(\lambda) = \lambda I - L(e^{\lambda\cdot}I),\ \lambda \in \mathbb{C}.$$

We shall also use the state-space $X = H^1(-r,0;R^n)$ for system (1.1). X endowed with the inner product

$$\langle\phi,\psi\rangle_X = \langle\phi(0),\psi(0)\rangle_{R^n} + \langle\dot{\phi},\dot{\psi}\rangle_{L^2}$$

is metricaloly isomorphis to dom $\mathcal{A}$ endowed with the graph norm, which is a Hilbert space, because $\mathcal{A}$ is closed. The isomorphism dom $\mathcal{A} \to X$ is given by

$$\iota(\phi(0),\phi) = \phi,\ (\phi(0),\phi) \in \text{dom } \mathcal{A},$$

$$\iota^{-1}\phi = (\phi(0),\phi),\ \phi \in X. \tag{1.5}$$

In a recent paper [7] I. Lasiecka and A. Manitius gave for the first time optimal rates for the AV-scheme considered in [1]. These estimates are essentially based on uniform (with respect to the approximation parameter) differentiability of the approximation semigroups, which means that the characterization of differentiable semigroups in [8; Theorem 4.7] is uniformly valid for the approximating semigroups.

In this paper we develope a scheme using first order splines which essentially has all the good properties of the AV-scheme. In addition we are able to establish error estimates analogous to those in [7]. As expected we obtain quadratic convergence for sufficiently smooth data. Naturally uniform differentiability is also the essential basis for our approach which as far as convergence rates are concerned is motivated by [7].

§2. Spline approximation.

Let $t_k^N = -k\frac{r}{N}$, $k = 0,\cdots,N$, and $t_{-1}^N = 0$, $t_{N+1}^N = -r$ for $N = 1,2,\cdots$. B_k^N, $k = 0,\cdots,N$, denotes the usual first order B-splines on the interval $[-r,0]$ corresponding to the mesh $t_0^N,\cdots,t_N^N$,

$$B_k^N(\theta) = \begin{cases} \frac{N}{r}(\theta - t_{k+1}^N) & \text{for } t_{k+1}^N \leq \theta \leq t_k^N, \\ \frac{N}{r}(t_{k-1}^N - \theta) & \text{for } t_k^N \leq \theta \leq t_{k-1}^N, \\ 0 & \text{elsewhere.} \end{cases}$$

Furthermore, we put

$$E_k^N = \chi_{[t_k^N,t_{k-1}^N)},\ k = 1,\cdots,N,$$

and

$$\hat{E}_0^N = (I,0),\ \hat{E}_k^N = (0,E_k^N I),\ k = 1,\cdots,N,$$

where I denotes the $n\times n$ identity matrix. The following spaces will be used in the sequel:

$$W^N = \text{span}(E_1^N I,\cdots,E_N^N I) \subset L^2(-r,0;R^n),$$

$$Z^N = R^n\times W^N = \text{span}(\hat{E}_0^N,\cdots,\hat{E}_N^N) \subset Z,$$

$$X^N = \text{span}(B_0^N I,\cdots,B_N^N I) \subset X,$$

$$Z_1^N = \iota^{-1}X^N \subset \text{dom } \mathcal{A}.$$

It is convenient to introduce the "basis" matrices

$$E^N = (E_1^N I,\cdots,E_N^N I),$$

$$\hat{E}^N = (\hat{E}_0^N,\cdots,\hat{E}_N^N).$$

NASA Contractor Report 178342

ICASE REPORT NO. 87-47

ICASE

ON NON-CONVERGENCE OF ADJOINT SEMIGROUPS FOR CONTROL SYSTEMS WITH DELAYS

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Contract No. NAS1-18107
July 1987

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ON NON-CONVERGENCE OF ADJOINT SEMIGROUPS FOR CONTROL SYSTEMS WITH DELAYS+

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ABSTRACT

It is shown that the adjoints of a spline based approximation scheme for delay equations do not converge strongly.

+Part of this research was supported by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18107 while the authors were in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665.

[1]This work was supported in part by AFOSR Grant AFOSR-85-0287.

[2]This work was supported in part by AFOSR under Contract No. F-49620-86-C-011, by NASA under Grant No. NAG-1-517, and by NSF under Grant No. UINT-8521208.